The Solution

The second project consisted of four non linear pseudo range equations which define each satellite:

$$\begin{aligned}
\text{f1:} \quad & -(\mathrm{PR1} - cT)^2 + (x_1 - \mu_x)^2 + (y_1 - \mu_y)^2 + (z_1 - \mu_z)^2 &= 0 \\
\text{f2:} \quad & -(\mathrm{PR2} - cT)^2 + (x_2 - \mu_x)^2 + (y_2 - \mu_y)^2 + (z_2 - \mu_z)^2 &= 0 \\
\text{f3:} \quad & -(\mathrm{PR3} - cT)^2 + (x_3 - \mu_x)^2 + (y_3 - \mu_y)^2 + (z_3 - \mu_z)^2 &= 0 \\
\text{f4:} \quad & -(\mathrm{PR4} - cT)^2 + (x_4 - \mu_x)^2 + (y_4 - \mu_y)^2 + (z_4 - \mu_z)^2 &= 0
\end{aligned}$$

In these equations we are given the pseudo ranges (PR's) and the position of the satellite (x, y, z). However, the position of the satellites is given in Latitude and Longitude, so the first step in this problem was to write a MatLab program that calculated the appropriate x, y, and z coordinates of the satellite given the known Latitude, Longitude, and altitude of the satellite. Simply, here is that code:

```
i = 1;
while i<=4
    x(i) = R_Sat*cos(Lat_Sat(i)*pi/180)*cos(Lon_Sat(i)*pi/180);
    y(i) = R_Sat*cos(Lat_Sat(i)*pi/180)*sin(Lon_Sat(i)*pi/180);
    z(i) = R_Sat*sin(Lat_Sat(i)*pi/180);
    i = i+1;
end
```

Next, the four range equations were defined in Maple with the appropriate input parameters (x, y, z) determined by Matlab as follows:

```
> f1:=-(0-c*T)^2+(8.994017e6-ux)^2+(-1.714969e7-uy)^2
     +(1.820693e7-uz)^2;
> f2:=-(-.001587009*c-c*T)^2+(1.800745e6-ux)^2
     +(-2.155004e7-uy)^2+(1.545476e7-uz)^2;
> f3:=-(-.001373255*c-c*T)^2+(-.395791e6-ux)^2
     +(-2.323254e7-uy)^2+(1.290693e7-uz)^2;
> f4:=-(-.001092284*c-c*T)^2+(-1.762432e6-ux)^2
     +(-2.005038e7-uy)^2+(1.736009e7-uz)^2;
```

Lastly, the nonlinear equation solver (solve) was used to determine approximate parameter values that satisfy all of these equations (μ_x, μ_y, μ_z, T):

```
> solve({f1,f2,f3,f4},{ux,uy,uz,T})
```

The following two solutions were obtained:

$$\begin{aligned}
\mu_x &= -9.916456796 \cdot 10^5 \\
\mu_y &= 9.954431887 \cdot 10^6 \\
\mu_z &= -6.689125157 \cdot 10^6 \\
T &= 0.1271708655
\end{aligned}$$

and

$$
\begin{aligned}
\mu_x &= 5.193936983 \cdot 10^5 \\
\mu_y &= -5.278156916 \cdot 10^6 \\
\mu_z &= 3.546845993 \cdot 10^6 \\
T &= -0.06896617849
\end{aligned}
$$

Later we typed these numbers back in to each equation to find if all of the equations are satisfied by a single solution. We found that the equations were not satisfied exactly. Our understanding is that the solve command in Maple uses a least squares fit if no solution exists in the four dimensions of interest. There is some ambiguity because Maple produces two solutions for the set of nonlinear equations. However, the first solution was immediately thrown out because its calculated radius $r = \sqrt{\mu_x^2 + \mu_y^2 + \mu_z^2}$ is not close to that of the earth's radius. Lastly the Latitude and Longitude values were calculated from the μ_x, μ_y, and μ_z solutions produced by Maple and determined to be the following:

$$
\begin{aligned}
\text{Latitude} &= 33.7728^\circ \\
\text{Longitude} &= -84.3799^\circ
\end{aligned}
$$

Another issue worthy of noting is that initially the precision of the satellite positions was rounded in MatLab. This first set of solutions put us somewhere in south east Atlanta, clearly not the correct solution. Therefore, the values were outputted with higher precision which eventually put us exactly at the corner of Mytle St. and Ponce St.